SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22224

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any FIVE of the following:**	**10**
	a)	State whether the function $f(x)=\frac{e^x+e^{-x}}{2}$ is odd or even.	**02**
	Ans	$f(x)=\frac{e^x+e^{-x}}{2}$	
		$\therefore f(-x)=\frac{e^{-x}+e^{-(-x)}}{2}$	½
		$=\frac{e^{-x}+e^{x}}{2}$	½
		$=f(x)$	½
		$\therefore$ function is even.	½
	b)	If $f(x)=\frac{x^2+1}{x^3-1}$ find $f\left(\frac{1}{2}\right)$	**02**
	Ans	$f(x)=\frac{x^2+1}{x^3-1}$	
		$\therefore f\left(\frac{1}{2}\right)=\frac{\left(\frac{1}{2}\right)^2+1}{\left(\frac{1}{2}\right)^3-1}$	1
		$=\frac{-10}{7}$ OR -1.429	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	c)	Find $\frac{dy}{dx}$, if $y=\left(x^2+1\right)^5$	**02**
	Ans	$y=\left(x^2+1\right)^5$	
		$\therefore \frac{dy}{dx}=5\left(x^2+1\right)^4.\frac{d}{dx}\left(x^2+1\right)$	
		$=5\left(x^2+1\right)^4.(2x)$	1
		$=10x\left(x^2+1\right)^4$	1
	d)	Evaluate $\int(\tan x+\cot x)^2\ dx$	**02**
	Ans	$\int(\tan x+\cot x)^2\ dx$	
		$=\int\left(\tan^2 x+2\tan x\cot x+\cot^2 x\right)dx$	½
		$=\int\left(\tan^2 x+2+\cot^2 x\right)dx$	
		$=\int\left[\left(\sec^2 x-1\right)+2+\left(co\sec^2 x-1\right)\right]dx$	½
		$=\int\left(\sec^2 x+co\sec^2 x\right)dx$	
		$=\tan x-\cot x+c$	½ +½
	e)	Evaluate $\int\log x dx$	**02**
	Ans	$\int\log x dx=\int\log x\cdot 1\ dx$	½
		$=\log x\int 1dx-\int\left(\int 1dx\ \frac{d}{dx}\log x\right)dx$	½
		$=\log x(x)-\int x\ \frac{1}{x}dx$	½
		$=x\log x-\int 1dx$	
		$=x\log x-x+c$	½
		$=x(\log x-1)+c$	
	f)	Find the area between the lines $y=3x$, x-axis and the ordinates $x=1$ and $x=5$	**02**
	Ans	Area $A=\int_a^b ydx$	
		$=\int_1^5 3xdx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	$= 3\int_1^5 x dx$	
		$= 3\left[\frac{x^2}{2}\right]_1^5$	½
		$= 3\left[\frac{5^2}{2} - \frac{1^2}{2}\right]$	½
		$= 36$	½
		---	
	g)	Show that there exist a root of the equation $x^2 - 2x - 1 = 0$ in $(-1, 0)$ and find approximate value of the root by using Bisection method. (Use two iterations)	**02**
	Ans	$x^2 - 2x - 1 = 0$	
		$f(x) = x^2 - 2x - 1$	
		$f(-1) = 2$	
		$f(0) = -1$	½
		root is in $(-1, 0)$	
		$\therefore x_1 = \frac{-1+0}{2} = -0.5$	½
		$\therefore f(-0.5) = 0.25$	½
		$\therefore$ root is in $(-0.5, 0)$	
		$\therefore x_2 = \frac{-0.5+0}{2} = -0.25$	½
		OR	
		$x^2 - 2x - 1 = 0$	
		$f(x) = x^2 - 2x - 1$	
		$f(-1) = 2$	
		$f(0) = -1$	
		root is in $(-1, 0)$	

a	b	$x = \frac{a+b}{2}$	$f(x)$	Marks
-1	0	-0.5	0.25	1
-0.5	0	-0.25	-----	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.		**Attempt any THREE of the following:**	**12**
	a)	Find $\frac{dy}{dx}$ if $\cos\left(x^2+y^2\right)=\log(xy)$	**04**
	Ans	$\cos\left(x^2+y^2\right)=\log(xy)$	
		$\therefore -\sin\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right)=\frac{1}{xy}\left(x\frac{dy}{dx}+y\right)$	2
		$\therefore -2x\sin\left(x^2+y^2\right)-2y\sin\left(x^2+y^2\right)\frac{dy}{dx}=\frac{1}{y}\frac{dy}{dx}+\frac{1}{x}$	
		$\therefore \frac{dy}{dx}\left(-2y\sin\left(x^2+y^2\right)-\frac{1}{y}\right)=\frac{1}{x}+2x\sin\left(x^2+y^2\right)$	1
		$\therefore \frac{dy}{dx}=\frac{\frac{1}{x}+2x\sin\left(x^2+y^2\right)}{-2y\sin\left(x^2+y^2\right)-\frac{1}{y}}$	1
		--	
	b)	If $x=a\cos^3\theta, y=a\sin^3\theta$, find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$	**04**
	Ans	$x=a\cos^3\theta$ $y=a\sin^3\theta$	
		$\therefore \frac{dx}{d\theta}=-3a\cos^2\theta\sin\theta$ $\frac{dy}{d\theta}=3a\sin^2\theta(\cos\theta)$	1
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$\frac{dy}{dx}=\frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$	1
		$\frac{dy}{dx}=-\tan\theta$	1
		at $\theta=\frac{\pi}{4}$	
		$\frac{dy}{dx}=-\tan\frac{\pi}{4}$	
		$\frac{dy}{dx}=-1$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	Find the maximum and minimum value of $2x^3-3x^2-36x+10$	04
	Ans	Let $y=2x^3-3x^2-36x+10$	
		$\therefore \frac{dy}{dx}=6x^2-6x-36$	½
		$\therefore \frac{d^2y}{dx^2}=12x-6$	½
		Consider $\frac{dy}{dx}=0$	
		$6x^2-6x-36=0$	½
		$x^2-x-6=0$	
		$\therefore x=-2\ ,\ x=3$	½
		at $x=-2$	
		$\frac{d^2y}{dx^2}=12(-2)-6=-30<0$	½
		$\therefore y$ is maximum at $x=-2$	
		$y_{\max}=2(-2)^3-3(-2)^2-36(-2)+10$	
		$=54$	½
		$at\ \ x=3$	
		$\frac{d^2y}{dx^2}=12(3)-6=30>0$	½
		$\therefore y$ is minimum at $x=3$	
		$y_{\min}=2(3)^3-3(3)^2-36(3)+10$	
		$=-71$	½
		--	
	d)	A beam is bent in the form of the curve $y=2\sin x-\sin 2x$ Find the radius of curvature of the beam at the point $x=\frac{\pi}{2}$	04
	Ans	$y=2\sin x-\sin 2x$	½
		$\therefore \frac{dy}{dx}=2\cos x-2\cos 2x$	
		$\therefore \frac{d^2y}{dx^2}=-2\sin x+4\sin 2x$	½
		$\therefore$ at $x=\frac{\pi}{2}$	
		$\frac{dy}{dx}=2\cos\left(\frac{\pi}{2}\right)-2\cos 2\left(\frac{\pi}{2}\right)=2$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	$\frac{d^2y}{dx^2} = -2\sin\left(\frac{\pi}{2}\right) + 4\sin 2\left(\frac{\pi}{2}\right) = -2$	½
		$\therefore$ Radius of curvature is $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho = \frac{\left[1+(2)^2\right]^{\frac{3}{2}}}{-2}$	1
		$\therefore \rho = -5.590$ or 5.590	1
3.		**Attempt any THREE of the following:**	**12**
	a)	Find the points on the curve $y = x^3 + 3x^2 - 9x + 7$ at which tangents drawn are parallel to $x-$axis.	**04**
	Ans	$y = x^3 + 3x^2 - 9x + 7$	
		$\frac{dy}{dx} = 3x^2 + 6x - 9$	1
		$\because$ tangent is parallel to x-axis	
		$\therefore$ slope of tangent = slope of x-axis	
		$\therefore \frac{dy}{dx} = 0$	1
		$\therefore 3x^2 + 6x - 9 = 0$	
		$\therefore x = 1 \;;\; x = -3$	
		$\therefore y = 2 \;;\; y = 34$	1
		$\therefore$ points are $(1,2)$ and $(-3,34)$	1
	b)	Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$	**04**
	Ans	Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$	
		Put $x = \tan\theta \Rightarrow \tan^{-1}x = \theta$	
		$u = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$	½
		$u = \tan^{-1}(\tan 2\theta)$	½
		$u = 2\theta$	
		$u = 2\tan^{-1}x$	

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3.	b)	$\frac{du}{dx}=\frac{2}{1+x^2}$	½
		$v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$	
		$v=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$	½
		$v=\sin^{-1}\left(\sin 2\theta\right)$	½
		$v=2\theta$	
		$v=2\tan^{-1}x$	
		$\frac{dv}{dx}=\frac{2}{1+x^2}$	½
		$\therefore\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{2}{1+x^2}}{\frac{2}{1+x^2}}$	½
		$\therefore\frac{du}{dv}=1$	½
		OR	
		Let $u=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$	
		$\therefore\frac{du}{dx}=\frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2}\times\left[\frac{\left(1-x^2\right)2-2x\left(-2x\right)}{\left(1-x^2\right)^2}\right]$	1
		$\therefore\frac{du}{dx}=\frac{\left(1-x^2\right)^2}{\left(1-x^2\right)^2+4x^2}\left[\frac{2+2x^2}{\left(1-x^2\right)^2}\right]$	
		$\therefore\frac{du}{dx}=\frac{2+2x^2}{\left(1-x^2\right)^2+4x^2}$	
		$\therefore\frac{du}{dx}=\frac{2\left(1+x^2\right)}{\left(1-x^2\right)^2+4x^2}$	
		$\therefore\frac{du}{dx}=\frac{2\left(1+x^2\right)}{\left(1+x^2\right)^2}=\frac{2}{1+x^2}$	½
		$\therefore v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$\therefore \frac{dv}{dx}=\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}\times\left[\frac{(1+x^2)2-2x(2x)}{(1+x^2)^2}\right]$	1
		$\therefore \frac{dv}{dx}=\frac{(1+x^2)}{\sqrt{(1+x^2)^2-4x^2}}\left[\frac{2-2x^2}{(1+x^2)^2}\right]$	
		$\therefore \frac{dv}{dx}=\frac{(2-2x^2)}{(1+x^2)\sqrt{(1+x^2)^2-4x^2}}$	
		$\therefore \frac{dv}{dx}=\frac{2(1-x^2)}{(1+x^2)(1-x^2)}$	
		$\therefore \frac{dv}{dx}=\frac{2}{(1+x^2)}$	½
		$\therefore \frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{2}{(1+x^2)}}{\frac{2}{(1+x^2)}}$	½
		$\therefore \frac{du}{dv}=1$	½
	c)	Find $\frac{dy}{dx}$ if $y=(\log x)^x+x^{\cos^{-1}x}$	**04**
	Ans	Let $u=(\log x)^x$	
		$\log u = x\log(\log x)$	½
		$\frac{1}{u}\frac{du}{dx}=x\frac{1}{\log x}\frac{1}{x}+\log(\log x)$	1
		$\therefore \frac{du}{dx}=u\left(\frac{1}{\log x}+\log(\log x)\right)$	
		$\therefore \frac{du}{dx}=(\log x)^x\left[\frac{1}{\log x}+\log(\log x)\right]$	½
		Let $v=x^{\cos^{-1}x}$	
		$\log v=\cos^{-1}x\log x$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	$\frac{1}{v}\frac{dv}{dx}=\cos^{-1}x\left(\frac{1}{x}\right)+\log x\left(\frac{-1}{\sqrt{1-x^2}}\right)$	½
		$\therefore \frac{dv}{dx}=x^{\cos^{-1}x}\left[\left(\cos^{-1}x\right)\left(\frac{1}{x}\right)-\log x\left(\frac{1}{\sqrt{1-x^2}}\right)\right]$	½
		$\therefore \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$	
		$\therefore \frac{dy}{dx}=\left(\log x\right)^x\left[\frac{1}{\log x}+\log\left(\log x\right)\right]+x^{\cos^{-1}x}\left[\left(\cos^{-1}x\right)\left(\frac{1}{x}\right)-\log x\left(\frac{1}{\sqrt{1-x^2}}\right)\right]$	½
		--	
	d)	Evaluate: $\int\frac{\sec x\cos ecx}{\log\tan x}dx$	**04**
	Ans	$\int\frac{\sec x\cos ecx}{\log\tan x}dx$	
		Put $\log\tan x=t$	1
		$\therefore \frac{1}{\tan x}\sec^2 xdx=dt$	
		$\therefore \frac{\cos x}{\sin x}\frac{1}{\cos^2 x}dx=dt$	
		$\therefore \sec x\cos ecxdx=dt$	1
		$=\int\frac{1}{t}dt$	½
		$=\log t+c$	1
		$=\log\left(\log\left(\tan x\right)\right)+c$	½
		--	
4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate : $\int\frac{1}{2x^2+3x+1}dx$	**04**
	Ans	$\int\frac{1}{2x^2+3x+1}dx$	
		$=\frac{1}{2}\int\frac{1}{x^2+\frac{3}{2}x+\frac{1}{2}}dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	Third term $=\left(\frac{1}{2}\times\frac{3}{2}\right)^2=\frac{9}{16}$	
		$=\frac{1}{2}\int\frac{1}{x^2+\frac{3}{2}x+\frac{9}{16}-\frac{9}{16}+\frac{1}{2}}dx$	1
		$=\frac{1}{2}\int\frac{1}{\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$	1
		$=\frac{1}{2}\left[\frac{1}{2\left(\frac{1}{4}\right)}\log\left(\frac{x+\frac{3}{4}-\frac{1}{4}}{x+\frac{3}{4}+\frac{1}{4}}\right)\right]+c$	1½
		$=\log\left(\frac{2x+1}{2x+2}\right)+c$	
		OR	
		$\int\frac{1}{2x^2+3x+1}dx=\int\frac{1}{(2x+1)(x+1)}dx$	½
		Let $\frac{1}{(2x+1)(x+1)}=\frac{A}{2x+1}+\frac{B}{x+1}$	
		$1=A(x+1)+B(2x+1)$	
		Put $x=\frac{-1}{2}$	
		$\therefore A=2$	½
		Put $x=-1$	
		$\therefore B=-1$	½
		$\frac{1}{(2x+1)(x+1)}=\frac{2}{2x+1}+\frac{-1}{x+1}$	½
		$\int\frac{1}{(2x+1)(x+1)}dx=\int\left(\frac{2}{2x+1}+\frac{-1}{x+1}\right)dx$	
		$=\frac{2\log(2x+1)}{2}-\log(x+1)+c$	1+1
		$=\log(2x+1)-\log(x+1)+c$	
		OR	

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4.	a)	$\int \frac{1}{2x^2+3x+1}dx$	
		Third term $=\frac{(M.T.)^2}{4(F.T.)}=\frac{9}{8}$	½
		$=\int \frac{1}{2x^2+3x+\frac{9}{8}-\frac{9}{8}+1}dx$	1
		$=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\frac{1}{8}}dx$	1
		$=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\left(\frac{1}{\sqrt{8}}\right)^2}dx$	½
		$=\frac{1}{\sqrt{2}}\left[\frac{1}{2\left(\frac{1}{\sqrt{8}}\right)}\log\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{8}}-\frac{1}{\sqrt{8}}}{\sqrt{2}x+\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{8}}}\right)\right]+c$	1
		$=\log\left(\frac{2x+1}{2x+2}\right)+c$	
		--	
	b)	Evaluate : $\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$	**04**
	Ans	$\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$	
		$=\int \frac{dx/\cos^2 x}{\frac{a^2\sin^2 x+b^2\cos^2 x}{\cos^2 x}}$	½
		$=\int \frac{\sec^2 x\,dx}{a^2\tan^2 x+b^2}$ Put $\tan x=t$ $\therefore \sec^2 x\,dx=dt$	½
		$=\int \frac{dt}{a^2 t^2+b^2}$	1
		$=\int \frac{dt}{a^2\left(t^2+\frac{b^2}{a^2}\right)}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$=\frac{1}{a^2}\int\frac{dt}{t^2+\left(\frac{b}{a}\right)^2}=\frac{1}{a^2}\cdot\frac{1}{\frac{b}{a}}\tan^{-1}\left(\frac{t}{\frac{b}{a}}\right)+c$	1
		$=\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$	½
		OR	
		$\int\frac{dx}{a^2\sin^2x+b^2\cos^2x}$	
		$=\int\frac{dx/\cos^2x}{\frac{a^2\sin^2x+b^2\cos^2x}{\cos^2x}}$	½
		$=\int\frac{\sec^2 xdx}{a^2\tan^2x+b^2}$ $Put\ \tan x=t$, $\therefore\sec^2 xdx=dt$	½
		$=\int\frac{dt}{a^2t^2+b^2}$	1
		$=\frac{1}{b}\tan^{-1}\left(\frac{at}{b}\right)\frac{1}{a}+c$	1½
		$=\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$	½
	c)	Evaluate : $\int x\operatorname{cosec}^{-1}xdx$	**04**
	Ans	$\int x\operatorname{cosec}^{-1}xdx$	
		$=\operatorname{cosec}^{-1}x\int xdx-\int\left(\int xdx\ \frac{d}{dx}\operatorname{cosec}^{-1}x\right)dx$	½
		$=\operatorname{cosec}^{-1}x\cdot\frac{x^2}{2}-\int\frac{x^2}{2}\cdot\left(\frac{-1}{x\sqrt{x^2-1}}\right)\cdot dx$	1
		$=\operatorname{cosec}^{-1}x\cdot\frac{x^2}{2}+\frac{1}{2}\int\frac{x}{\sqrt{x^2-1}}\cdot dx$	
		$=\operatorname{cosec}^{-1}x\cdot\frac{x^2}{2}+\frac{1}{4}\int\frac{2x}{\sqrt{x^2-1}}\cdot dx$	1
		$=\operatorname{cosec}^{-1}x\cdot\frac{x^2}{2}+\frac{1}{4}\left(2\sqrt{x^2-1}\right)+c$	1
		$=\operatorname{cosec}^{-1}x\cdot\frac{x^2}{2}+\frac{1}{2}\left(\sqrt{x^2-1}\right)+c$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	Evaluate : $\int \frac{1}{x(2-\log x)(2\log x-1)}dx$	04
	Ans	$\int \frac{1}{x(2-\log x)(2\log x-1)}dx$ Put $\log x = t$, $\therefore \frac{1}{x}dx = dt$	½
		$\int \frac{1}{(2-t)(2t-1)}dt$	½
		$\frac{1}{(2-t)(2t-1)} = \frac{A}{2-t} + \frac{B}{2t-1}$	
		$1 = A(2t-1) + B(2-t)$	
		$\therefore$ Put $t=2$, $A = \frac{1}{3}$	½
		Put $t = \frac{1}{2}$, $B = \frac{2}{3}$	½
		$\therefore \frac{1}{(2-t)(2t-1)} = \frac{\frac{1}{3}}{2-t} + \frac{\frac{2}{3}}{2t-1}$	
		$\int \frac{1}{(2-t)(2t-1)}dt = \int \left(\frac{\frac{1}{3}}{2-t} + \frac{\frac{2}{3}}{2t-1} \right) dt$	½
		$= -\frac{1}{3}\log[2-t] + \frac{2}{6}\log[2t-1] + c$	1
		$= -\frac{1}{3}\log[2-\log x] + \frac{1}{3}\log[2\log x - 1] + c$	½
		OR	
		$\int \frac{1}{x(2-\log x)(2\log x-1)}dx$ Put $\log x = t$, $\therefore \frac{1}{x}dx = dt$	½
		$\int \frac{1}{(2-t)(2t-1)}dt$	
		$= \int \frac{1}{-2t^2+5t-2}dt$	
		$= \frac{-1}{2}\int \frac{1}{t^2 - \frac{5}{2}t + 1}dt$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	$= \frac{-1}{2}\int \frac{1}{t^2 - \frac{5}{2}t + \frac{25}{16} - \frac{25}{16} + 1} dt$	½
		$= \frac{-1}{2}\int \frac{1}{\left(t-\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2} dt$	1
		$= \frac{-1}{2}\frac{1}{2\frac{3}{4}}\log\left\|\frac{t-\frac{5}{4}-\frac{3}{4}}{t-\frac{5}{4}+\frac{3}{4}}\right\| + c$	1
		$= \frac{-1}{3}\log\left\|\frac{t-2}{t-\frac{1}{2}}\right\| + c$	
		$= \frac{-1}{3}\log\left\|\frac{\log x - 2}{\log x - \frac{1}{2}}\right\| + c$	½
		--	
	e)	Evaluate: $\int_1^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}} dx$	**04**
	Ans	$I = \int_1^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}} dx$ ---------------------(1)	
		$I = \int_1^4 \frac{\sqrt[3]{9-(5-x)}}{\sqrt[3]{9-(5-x)}+\sqrt[3]{(5-x)+4}} dx$	1
		$\therefore I = \int_1^4 \frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4}+\sqrt[3]{9-x}} dx$---------------------(2)	
		add (1) and (2), $I + I = \int_1^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}} dx + \int_1^4 \frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4}+\sqrt[3]{9-x}} dx$	
		$\therefore 2I = \int_1^4 \frac{\sqrt[3]{9-x}+\sqrt[3]{x+4}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}} dx$	1
		$\therefore 2I = \int_1^4 1\, dx$	
		$\therefore 2I = (x)_1^4$	1
		$\therefore I = \frac{3}{2}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.		**Attempt any TWO of the following:**	**12**
	a)	Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{9}+\frac{y^2}{4}$ about $x-$axis	**06**
	Ans	Consider $\frac{x^2}{9}+\frac{y^2}{4}=1$	
		$\therefore y^2=\frac{4}{9}\left(9-x^2\right)$	1
		Volume of solid $V=\pi\int_{-a}^{a} y^2 dx$	
		$V=\pi\int_{-3}^{3}\frac{4}{9}\left(9-x^2\right)dx$	1
		$\therefore V=2\pi\int_{0}^{3}\frac{4}{9}\left(9-x^2\right)dx$	1
		$\therefore V=\frac{8\pi}{9}\left[9x-\frac{x^3}{3}\right]_0^3$	1
		$\therefore V=\frac{8\pi}{9}\left[\left(9(3)-\frac{3^3}{3}\right)-\left(9(0)-\frac{0^3}{3}\right)\right]$	1
		$V=16\pi$	1
		(Note :If student has considered/assumed other value than 1and attempted to solve the problem , give appropriate marks.)	
		--	
	b)	**Attempt the following:**	**06**
	(i)	Form the diffrential equation by eliminating the arbitrary constants if $y= a\cos\left(\log x\right)+b\sin\left(\log x\right)$	**03**
	Ans	$y= a\cos\left(\log x\right)+b\sin\left(\log x\right)$	
		$\therefore \frac{dy}{dx}=-a\sin\left(\log x\right)\frac{1}{x}+b\cos\left(\log x\right)\frac{1}{x}$	1
		$\therefore x\frac{dy}{dx}=-a\sin\left(\log x\right)+b\cos\left(\log x\right)$	
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-a\cos\left(\log x\right)\frac{1}{x}-b\sin\left(\log x\right)\frac{1}{x}$	1
		$\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-\left(a\cos\left(\log x\right)+b\sin\left(\log x\right)\right)$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)(i)	$\therefore x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=-y$	½
		$\therefore x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$	
	b)(ii)	Solve the differential equation: $\dfrac{dy}{dx}+y\tan x=\cos^2 x$	**03**
	Ans	$\dfrac{dy}{dx}+y\tan x=\cos^2 x$ Comparing with $\dfrac{dy}{dx}+Py=Q$	½
		$\therefore P=\tan x$ and $Q=\cos^2 x$	
		$IF=e^{\int\tan x dx}=e^{\log(\sec x)}=\sec x$	½
		$\therefore y\cdot IF=\int Q\cdot IF dx+c$	
		$y\cdot\sec x=\int\cos^2 x\sec x dx+c$	1
		$y\cdot\sec x=\int\cos x dx+c$	
		$y\cdot\sec x=\sin x+c$	1
	c)	In a single closed electrical circuit the current ' I ' at time t is given by $E-RI-L\dfrac{dI}{dt}=0$. Find the current I at time t, given that t=0 , I=0 and L,R,E are constants.	**06**
	Ans	$E-RI-L\dfrac{dI}{dt}=0$	
		$\therefore \dfrac{dI}{dt}+\dfrac{R}{L}I=\dfrac{E}{L}$ Comparing with $\dfrac{dy}{dx}+Py=Q$	½
		$\therefore P=\dfrac{R}{L}$ and $Q=\dfrac{E}{L}$	
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore I\cdot IF=\int Q\cdot IF dt+c$	
		$I\cdot e^{\frac{R}{L}t}=\int\dfrac{E}{L}e^{\frac{R}{L}t}dt+c$	1
		$I\cdot e^{\frac{R}{L}t}=\dfrac{E}{L}\dfrac{e^{\frac{R}{L}t}}{\frac{R}{L}}+c$	1
		$I\cdot e^{\frac{R}{L}t}=\dfrac{E}{R}e^{\frac{R}{L}t}+c$	
		When $t=0$, $\mathrm{I}=0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$\therefore c = -\frac{E}{R}$	1
		$I \cdot e^{\frac{R}{L}t} = \frac{E}{R} e^{\frac{R}{L}t} - \frac{E}{R}$	½
		$I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$	1
		--	
6.		**Attempt any TWO of the following:**	**12**
	a)	**Attempt the following:**	**06**
	(i)	Solve the following system of by equtions by Jacobi's -Iteration method. $\left(\text{Two iterations}\right)$	**03**
		$5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$	
	Ans	$x = \frac{1}{5}(12 - 2y - z)$	
		$y = \frac{1}{4}(15 - x - 2z)$	
		$z = \frac{1}{5}(20 - x - 2y)$	1
		Starting with $x_0 = y_0 = z_0 = 0$	
		$x_1 = 2.4$	
		$y_1 = 3.75$	
		$z_1 = 4$	1
		$x_2 = 0.1$	
		$y_2 = 1.15$	
		$z_2 = 2.02$	1
		--	
	a(ii)	Solve the following system of equation by using Gauss-Seidel method. $\left(\text{Two iterations}\right)$	**03**
		$15x + 2y + z = 18$, $2x + 20y - 3z = 19$, $3x - 6y + 25z = 22$	
	Ans	$x = \frac{1}{15}(18 - 2y - z)$	
		$y = \frac{1}{20}(19 - 2x + 3z)$	
		$z = \frac{1}{25}(22 - 3x + 6y)$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a)(ii)	Starting with $y_0 = z_0 = 0$ $x_1 = 1.2$ $y_1 = 0.83$ $z_1 = 0.935$	1
		$x_2 = 1.027$ $y_2 = 0.988$ $z_2 = 0.994$	1
	b)	Solve the following system of equations by using Gauss-elimination method $6x - y - z = 19$, $3x + 4y + z = 26$, $x + 2y + 6z = 22$	**06**
	Ans	$6x - y - z = 19$ $3x + 4y + z = 26$ $x + 2y + 6z = 22$	
		$6x - y - z = 19$, $3x + 4y + z = 26$; + : $9x + 3y = 45$ and $18x + 24y + 6z = 156$, $x + 2y + 6z = 22$; − : $17x + 22y = 134$	1+1
		$\therefore\ 3x + y = 15$ $66x + 22y = 330$ $17x + 22y = 134$ − : $49x = 196$	
		$\therefore x = 4$	1
		$y = 3$	1
		$z = 2$	1
		Note: *In the above solution, first z is eliminated and then y is eliminated to find the value of x first. If in case the problem is solved by elimination of another unknown i. e., either first x or y, appropriate marks to be given as per above scheme of marking.*	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	Find the approximate root of the equation $x^4-x-10=0$, by Newton-Raphson method (Carry out four iterations)	
	Ans	Let $f(x)=x^4-x-10$	
		$f(1)=-10<0$	
		$f(2)=4>0$	1
		$f'(x)=4x^3-1$	1
		Initial root $x_0=2$	
		$\therefore f'(2)=31$	
		$x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=2-\dfrac{f(2)}{f'(2)}=1.871$	1
		$x_2=1.871-\dfrac{f(1.871)}{f'(1.871)}=1.856$	1
		$x_3=1.856-\dfrac{f(1.856)}{f'(1.856)}=1.856$	1
		$x_4=1.856-\dfrac{f(1.856)}{f'(1.856)}=1.856$	1
		OR	
		Let $f(x)=x^4-x-10$	
		$f(1)=-10<0$	
		$f(2)=4>0$	1
		$f'(x)=4x^3-1$	1
		Initial root $x_0=2$	
		$\therefore f'(2)=31$	
		$x_i=\dfrac{xf'(x)-[f(x)]}{f'(x)}=\dfrac{x(4x^3-1)-[x^4-x-10]}{4x^3-1}$	
		$=\dfrac{3x^4+10}{4x^3-1}$	2
		$x_1=1.871$	½
		$x_2=1.856$	½
		$x_3=1.856$	½
		$x_4=1.856$	½
		OR	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	Let $f(x)=x^4-x-10$	
		$f(-1)=-8<0$	
		$f(-2)=8>0$	1
		$f'(x)=4x^3-1$	1
		Initial root $x_0=-2$	
		$\therefore f'(-2)=-33$	
		$x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=-2-\dfrac{f(-2)}{f'(-2)}=-1.758$	1
		$x_2=-1.758-\dfrac{f(-1.758)}{f'(-1.758)}=-1.700$	1
		$x_3=-1.700-\dfrac{f(-1.700)}{f'(-1.700)}=-1.697$	1
		$x_4=-1.697-\dfrac{f(-1.697)}{f'(-1.697)}=-1.697$	1
		OR	
		Let $f(x)=x^4-x-10$	
		$f(-1)=-8<0$	
		$f(-2)=8>0$	1
		$f'(x)=4x^3-1$	1
		Initial root $x_0=-2$	
		$\therefore f'(-2)=-33$	
		$x_i=\dfrac{xf'(x)-[f(x)]}{f'(x)}=\dfrac{x(4x^3-1)-[x^4-x-10]}{4x^3-1}$	
		$=\dfrac{3x^4+10}{4x^3-1}$	2
		$x_1=-1.758$	½
		$x_2=-1.700$	½
		$x_3=-1.697$	½
		$x_4=-1.697$	½
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SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* -- --	